Spatiotemporal data

ST733 – Spatial Statistics

Spatiotemporal Data

- Types of spatiotemporal data:
- Examples of each type:

Spatiotemporal Data

- Goals of a spatiotemporal analysis:

- "Spatiotemporal", "Spatio-temporal" or "Spatial-temporal"?

Discrete time, time series models

- The simplest interesting times series model is the first-orderautoregressive (AR1) model:
- The process is stationary :

Discrete time, time series models

- The AR1 is equivalent to an exponential spatial correlation model in 1D

Discrete time, time series models

- We'll stick with the AR1 case, but this is just a special case of the ARMA(p, q) model:

Continuous space, discrete time

- The spatial AR1 model is:
- The mean and covariance are:
- This is an example of separability

Separability

- A correlation function is separable if

$$\text{Cor}[Y_{t_1}(\mathbf{s}_1), Y_{t_2}(\mathbf{s}_2)] = C_1(\mathbf{s}_1, \mathbf{s}_2)C_2(t_1, t_2)$$

- This implies (a) the spatial correlation is same for all times and (b) the temporal correlation is same for all locations

- Separability leads to dramatic computation savings assuming the same locations ($\mathbf{s} \in \{\mathbf{s}_1, ..., \mathbf{s}_n\}$) are measured every time step ($t \in \{1, ..., m\}$)

Separability

- Let $\mathbf{Y}_t = [Y_t(\mathbf{s}_1), ..., Y_t(\mathbf{s}_n)]^T$ and $\mathbf{Y} = (\mathbf{Y}_1^T, ..., \mathbf{Y}_m^T)$
- Y is a spatiotemporal GP with mean $\mathrm{E}[Y_t(\mathbf{s})] = \mathbf{X}_t(\mathbf{s})\beta$ and separable covariance:
- So the joint distribution of $\mathbf{Y}$ can be written:

Separability

- The marginal mean and covariance of $\mathbf{Y}_t$ are:

- The mean and covariance of $\mathbf{Y}(\mathbf{s}) = [Y_1(\mathbf{s}), ..., Y_m(\mathbf{s})]$ are:

Seperability

- Facts about the Kronecker product:

Seperability

- Using these facts, the likelihood becomes:

- MLE and Bayes are fast if n and m are reasonably small

- Prediction, both spatial interpolation (Kriging) and forecasting ahead in time, follows from the conditional normal distributions

Continuous space, discrete time

- How to test for separability?

More flexible models

- As with stationarity and isotropy, separability is usually unrealistic but leads to huge simplifications

- Nonseparable models are easy to specify, but hard to fit

More flexible models

- Dynamic spatial linear model (DLM):

Continuous space/continuous time

- $Y(\mathbf{s}, t)$ is a GP with mean $\mathbf{X}(\mathbf{s}, t)\beta$ and

$$\text{Cov}[Y(\mathbf{s}, t), Y(\mathbf{s}', t')] = C(\mathbf{s}, \mathbf{s}', t, t')$$

- Typically we assume separability

$$C(\mathbf{s}, \mathbf{s}', t, t') = C_1(\mathbf{s}, \mathbf{s}')C_2(\mathbf{s}, \mathbf{s}')$$

- For example,

- The likelihood is MVN with a Kronecker product for the covariance

Continuous space/continuous time

- Is the separable ST covariance valid?
- We we think of $(\mathbf{s}, t)$ as a 3D space then any valid GP is 3D is valid
- Spatiotemporal spectral densities:
- Fact, if the spectral density is separable then the covariance is separable:

Continuous space/continuous time

- Non-separable covariances (Cressie & Huang; Gneiting):